Math 112 (Gordan Text) Name_____

Worksheet 12A Date_____

1. Classify each of the following difference equations according to the following categories: i) linear or non-linear; ii) order; iii) homogeneous or non-homogeneous by checking or filling in the appropriate box:

$a)\ x_{n+1} + n^2 + x_{n-1} = 0$

i) linear ___ non-linear___; ii) order ___; iii) homogeneous__ non-homogeneous__.

$b)\ x_{n+1} = \pi\ x_n - 2x_n{}^2$

i) linear ___ non-linear___; ii) order ___; iii) homogeneous__ non-homogeneous__.

c) $a_{n+1} = 1.5\ a_n - 6$

i) linear ___ non-linear___; ii) order ___; iii) homogeneous__ non-homogeneous__.

d) $y_{n+1} = \frac{1}{2}(\frac{1}{y_n} + y_n)$

i) linear ___ non-linear___; ii) order ___; iii) homogeneous__ non-homogeneous__.

2. a. Calculate by hand the first 6 terms of the sequence $\{x_n\}$ defined by the difference equation and initial value:

$$x_{n+1} = \frac{1}{2}x_n - 1 \qquad x_0 = 0$$

c

n	0	1	2	3	4	5	6
x_n							

b. Find the complete (i.e. general) solution of the homogeneous difference equation

$$x_{n+1} = (\frac{1}{2})x_n$$

associated with the non-homogeneous equation in Part a.

c. Find a particular solution $\{p_n\}$ of the difference equation $x_{n+1} = \frac{1}{2}x_n - 1 \qquad x_0 = 0$ in Part a. Show your work.

d. Use Parts b and c to find the solution $\{x_n\}$ of

$$x_{n+1} = \frac{1}{2}x_n - 1 \qquad x_0 = 0$$

Compare this solution with the tabular values in Part a.

3. a. Calculate by hand the first 6 terms of the sequence $\{x_n\}$ defined by the difference equation and initial value:

$$x_{n+1} = (\frac{1}{2})x_n + n \qquad x_0 = 0$$

n	0	1	2	3	4	5	6
x_n							

b. Find a particular solution $\{p_n\}$ of the difference equation $x_{n+1} = (\frac{1}{2})x_n + n \qquad x_0 = 0$ in Part a. Show your work.

c. Use Part b to find the solution $\{x_n\}$ of

$$x_{n+1} = (.5)x_n + n \qquad x_0 = 0$$

Compare this solution with your tabular values in Part a.

4. a. Calculate by hand the first 6 terms of the sequence $\{x_n\}$ defined by the difference equation and initial value:

n	0	1	2	3	4	5	6
x_n							

$$x_{n+1} = (\frac{1}{2})x_n + 10(\frac{1}{4})^n \qquad x_0 = 100$$

b. Find a particular solution $\{p_n\}$ of the difference equation $x_{n+1} = (\frac{1}{2})x_n + 10(\frac{1}{4})^n$ in Part a. Show your work.

5. Your grandmother is 65 and at the end of this year will have a retirement nest egg of $300,000 invested in a very secure account that pays 6% interest per year interest. She would like to withdraw $25,000 at the end of next year to supplement her social security income, and then to increase that end-of the-year withdrawal by 3% per year to account for inflation.

a. Write a difference equation and initial value for the balance B_n in her retirement account at the end of the nth-year.

b. Compute the year-end balances to the nearest 10 dollars up through the 6^{th}-year and record your results in the table below.

n	0	1	2	3	4	5	6
B_n							

c. Find the solution (i.e. the n^{th}-term formula) for the sequence $\{B_n\}$ of year-end balances. Use this formula to determine how many years Grandma's nest egg will last. Show your work.

6. A family plans to get a $100,000 mortgage at 6% annual interest rate over 30 years in order to purchase a new home.

a) What will the monthly payment be on this loan? Show your work.

b) What will be the total cost of the loan? Show your work.

c) Suppose that the family decides that it can make a $10,000 larger down payment so that the mortgage will be for $90,000 and suppose that they are able to get an interest rate of 5.5% for the 30 year mortgage from their credit union. How much difference will this new mortgage make in their monthly payments and in the total cost of the loan? Show your work.